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A GENERIC TWO-EQUATION TURBULENCE MODEL FOR GEOPHYSICAL APPLICATIONS

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We present a generic two-equation model for modeling turbulent mixing in the ocean. The model can simulate any of the existing length scale formulations. We have also taken this opportunity to incorporate recent developments in turbulence modeling into this model.

1. Introduction

There have been many developments in two-equation turbulence closure models, since the pioneering work of Mellor and Yamada (1982, MY henceforth) more than two decades ago, and the follow-on work by Galperin et al. (1988, GK henceforth) and Kantha and Clayson (1994, KC henceforth). Progress has been especially rapid in recent years. Based on a more complete formulation of the pressure covariances in the equations for second moments, Canuto et al. (2001b) and Cheng et al. (2003, CC henceforth) have derived an alternative form for the stability functions S_M and S_H in the algebraic expressions for momentum and scalar diffusivities resulting from second moment closure (see MY, CC or KC for details). These functions are an improvement, under unstable stratification, over the traditional Mellor and Yamada (1982) and Kantha and Clayson (1994) forms. Kantha (2003a, henceforth K, see also Kantha 2003b) has shown that despite the differences in the approach philosophy, similar results can be obtained by a slight retuning of the constants that appear in the KC (and MY) formulation.

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Burchard and Deleersnijder (2001, see also Burchard and Bolding 2001) have shown that the Canuto et al. (2001) formulation has less numerical noise than the original MY formulation of the stability functions. Burchard and Baumert (1995), Baumert and Peters (2000), Burchard (2001a, see also Burchard and Petersen 1999, Burchard and Bolding 2001) and Kantha (1988) have shown that the limitation on the length scale imposed by GK and KC under stable stratification can be supplanted by merely increasing the coefficient multiplying the buoyancy term in the length scale equation. Using LES, Cheng and Canuto (1994) have demonstrated the necessarily non-monotonic behavior of the turbulence length scale near the inversion in a PBL, which demonstrates the importance of either imposing the length scale limitation, under stable stratification, as GK and KC suggested, or modifying the constant multiplying the buoyancy term in the length scale equation to achieve the same effect. D'Alessio et al. (1998) and Burchard (2001b) have studied the effect of surface wave breaking on mixing in turbulence models, whereas Kantha and Clayson (2004) have parameterized both surface wave breaking and Langmuir turbulence in two-equation turbulence models. Mellor (2002) and Kantha (2004c) have investigated the role of surface gravity waves in determining the equivalent roughness scales in oscillatory bottom boundary layers.

The CARTUM team funded by the European Union has carefully examined the second moment closure methodology. A General Ocean

Turbulence Model (GOTM) has been constructed (Burchard et al. 1999) and made widely available to ocean turbulence modelers. GOTM includes a generic length scale formulation, with a conservation equation for the generic quantity $k^m \ell^n$ involving the turbulence length scale ℓ and turbulence kinetic energy (TKE) k, and can at present simulate the k-ε and k-ω turbulence models. Warner et al. (2004) have included this generic equation in a 3-D circulation model (ROMS) and Wijesekera et al. (2003) have examined the influence of turbulence models on modeled circulation in a 3-D ocean model.

In an effort to simulate the classic Dickey and Mellor (1980) experiments on grid-generated turbulence in stably-stratified fluids, Mellor (2001) suggested a Richardson number-dependent dissipation parameterization that modifies the traditional Prandtl-Kolmogorov formulation (this has been subsequently "expunged" by Mellor and Blumberg 2004). Baumert and Peters (2004a, 2004b) and Kantha (2004a, see also 2004b) have offered alternative models for explaining Dickey and Mellor (1980) observations on turbulent mixing in stably stratified fluids. For an excellent review of recent developments in oceanic mixed layer modeling, see Burchard (2002) and Baumert et al. (2004).

The TKE conservation equation can be readily derived from the Navier-Stokes equation, and the modeling of various terms in this equation has a firm theoretical basis. For example, the dissipation term is modeled making use of the fact that the dissipation in a turbulent fluid is independent of fluid viscosity and a function of only the energy-containing scales, with dissipative scales merely adjusting to the energy cascade down the wave number (frequency) spectrum, and the small scales near the high wave number end of the spectrum being isotropic. On the other hand, the physical basis for the conservation equation for some quantities involving the turbulence length scale is rather shaky, since this equation is often merely patterned after the TKE equation. Even when the equation can be derived rigorously from Navier-Stokes equations (as in the k-ε model), all the production and dissipation terms must be modeled. Kantha (2003c) has reviewed the length scale equation formulations and has shown that the diffusion term in the length scale equations must be modified to overcome many of the difficulties encountered by length scale modelers in the past. Kantha and Carniel (2003) have shown that with an appropriate combination of m and n, the generic length scale equation for $k^m \ell^n$ can be made to simulate any two-equation model including the traditional k-ε, k-ω and k-$k\ell$ and the non-traditional k-kT, k-T and k-ℓ models.

The above-mentioned developments suggest that the time is ripe to update the KC model. We have also taken this opportunity to formulate a generic two-equation turbulence model that can simulate any of the two-equation models mentioned above. We will demonstrate that the various subsets of the generic model produce nearly identical results. We will also investigate the changes that result from retuning of the closure constants (Kantha 2003c).

2. Update of the KC Model

The Kantha and Clayson (1994) two-equation turbulence model has been widely studied and their stability functions used in oceanic mixed layer simulations (for ex. Burchard 2002). In view of the recent developments [Kantha (2003a) and Burchard and colleagues (Burchard 2002)], it is time to update it. Consequently, we have investigated the changes brought on by a refinement of closure constants in the KC stability functions S_M and S_H in the mixing coefficients K_M and K_H for momentum and heat, respectively:

$$S_H = \frac{A_2\left(1 - \frac{6A_1}{B_1}\right)}{1 - 3A_2[6A_1 + B_2(1 - C_3)]G_H};$$

$$S_M = \frac{A_1\left(1 - \frac{6A_1}{B_1} - 3C_1\right) + 9A_1[2A_1 + A_2(1 - C_2)]S_H G_H}{1 - 9A_1 A_2 G_H} \tag{1}$$

where

$$K_M = q\ell S_M;\; K_H = q\ell S_H;\; G_H = \frac{\ell^2}{q^2}\beta g \frac{\partial \Theta}{\partial z}$$

In MY and KC models, A_1 = 0.92, B_1 = 16.6, C_1 = 0.08, A_2 = 0.74, B_2 = 10.1; C_2 = C_3 = 0 in MY and C_2 = 0.7 and C_3 = 0.2 in KC models. Kantha (2003a) has shown that the under prediction by MY and KC models of the Monin-Obukhov similarity function in the atmospheric surface layer for momentum under unstable stratification conditions can be overcome by a slight refinement of these closure constants: A_1 = 0.58, B_1 = 16.6, C_1 = 0.0384, A_2 = 0.62, B_2 = 12.04, C_2 = 0.429 and

C_3 = 0.2. Both KC and MY models use for the length scale equation:

$$\frac{\partial(q^2\ell)}{\partial t} + \frac{\partial}{\partial x_k}(U_k q^2\ell) = \frac{\partial}{\partial z}\left(S_\ell q\ell\frac{\partial(q^2\ell)}{\partial z}\right) + \frac{(q^2\ell)}{q^2}(E'_1P + E'_3B - E'_2\varepsilon\zeta) \tag{2}$$

where

$$\zeta = 1 + E'_5\left(\frac{\ell}{\kappa\ell_w}\right)^2$$

is the wall function needed to keep the turbulence diffusivity coefficient S_ℓ positive definite in the logarithmic region of a turbulent boundary layer (ℓ_w is the distance from the wall and κ is the von Karman constant). MY and KC chose $E'_1 = E'_3 =$ 1.8, E'_2 = 1, E'_5 = 1.33. Kantha and Clayson (2004) chose E'_5 = 4.8 to simulate surface wave breaking effects more accurately. KC and GK found it also necessary to bound the length scale under stable stratification by the Ozmidov length scale:

$$\ell < 0.53\frac{q}{N} \tag{3}$$

where N is the buoyancy frequency.

Following Kantha (2003c, 2004a), we update the constants in the length scale equation to: E'_1 = 2, E'_2 = 1, E'_3 = 5, E'_5 = 4.8. With the choice of numerical value much higher than unity for E'_3 under stable conditions, it is unnecessary to impose the limitation Eq. (3) on the length scale (Kantha 1988, Burchard and Baumert 1995, Baumert and Peters 2000, Burchard 2001a).

3. The Generic Model

The generic two-equation model proposed here is based on the work of Kantha and Carniel (2003, KC2 henceforth, see also Umlauf and Burchard 2003) to which the reader is referred for details. It consists of conservation equations for q^2 (twice the TKE) and $\phi = \psi^r \sim \left(q^{m'}\ell^{n'}\right)^r = q^m\ell^n$ a generic quantity involving the turbulence length scale ℓ (note the use of q^2 here instead of TKE, k, in KC2):

$$\begin{aligned}\frac{\partial q^2}{\partial t} + \frac{\partial}{\partial x_k}(U_k q^2) &= \frac{\partial}{\partial z}\left(\frac{S_q}{\sigma_k^\psi}q\ell\frac{\partial(q^2)}{\partial z}\right) + 2(P + P_L + B - \varepsilon)\\ \frac{\partial\varphi}{\partial t} + \frac{\partial}{\partial x_k}(U_k\varphi) &= \frac{\partial}{\partial z}\left(\frac{S_H}{\sigma_\Psi}q\ell\frac{\partial\varphi}{\partial z}\right) + \frac{\varphi}{q^2}(E_1P + E_4P_L + E_3B - E_2\varepsilon)\end{aligned} \tag{4}$$

where

$$P = q\ell S_M\frac{\partial U_k}{\partial z}\frac{\partial U_k}{\partial z}$$

is the shear production rate,

$$B = q\ell S_H\frac{g}{\rho_0}\frac{\partial\rho}{\partial z}$$

is the buoyancy production/ destruction rate, and

$$\varepsilon = \frac{q^3}{B_1\ell}$$

is the dissipation rate of TKE. σ_k^Ψ and σ_Ψ are Schmidt numbers; E_1, E_2, E_3, and E_4 are closure constants ($E_n = rC_{\Psi n}$); and P_L is the Langmuir turbulence production term given by $P = q\ell s_M\frac{\partial U_k}{\partial z}\frac{\partial V_k}{\partial z}$, where V_k is the Stokes velocity.

Eq. (4) is equivalent to a similar conservation equation for ψ (see KC2):

$$\frac{\partial\psi}{\partial t} + \frac{\partial}{\partial x_k}(U_k\psi) = \frac{\partial}{\partial z}\left(\frac{S_H}{\sigma_\psi}q\ell\frac{\partial\psi}{\partial z}\right) + \frac{S_H}{\sigma_\psi}q\ell\frac{(1-r)}{\psi}\left(\frac{\partial\psi}{\partial z}\right)^2 + \frac{\psi}{q^2}(c_{\psi1}P + c_{\psi4}P_L + c_{\psi3}G - c_{\psi2}\varepsilon) \tag{5}$$

but with the diffusion term modified by another term as shown. Note that there is no wall proximity function in either Eq. (4) or Eq. (5) thought to be necessary for making k-$k\ell$ model function properly (Umlauf and Burchard 2003).

The exponents m' and n' take particular values for the different turbulence length scale equations that have been used in the past: (i) ***k-ε*** model: m' = 3, n' = -1; (ii) ***k-ω*** model: m' = 1, n' = -1; (iii) ***k-kℓ*** model: m' = 2, n' = 1; (iv) ***k-kτ*** model: m' = 1, n' = 1; (v) ***k-ℓ*** model: m' = 0, n' = 1; (v) ***k-τ*** model: m' = -1, n' = 1. Quantity ω is the turbulence frequency and $\tau = k/\varepsilon$ is the turbulence time scale. It is possible to derive any of these length scale equations as a subset of a generic length scale equation. Table 1 gives values of the various parameters in Eqs. (4) and (5).

Table 1. Model parameters

	m	n	r	$c_{\psi1}$	$c_{\psi2}$	σ_k^Ψ	σ_Ψ
k-ε	2	-2/3	2/3	2	22/9	0.8	1.0667
k-ω	1/3	-1/3	1/3	1/3	5/9	0.8	0.5333
k-kℓ	-2/9	-1/9	-1/9	-2/9	-4/27	0.8	0.178
k-kτ	-1/2	-1/2	-1/2	-1/2	-1/6	0.8	0.8
k-ℓ	0	-1/3	-1/3	0	2/9	0.8	0.5333
k-τ	1/3	-1/3	-1/3	1/3	5/9	0.8	0.5333

4. Comparison with Observations

The updated KC model and the generic model are compared with observations. Following Burchard and Bolding (2001, see also Burchard 2002), we use wind-driven deepening of an initially linearly and stably stratified water column, the basis of the popular Kato and Phillips (1969, KP henceforth) study. KP performed a pioneering set of laboratory experiments on entrainment in an initially linearly-stratified fluid. A screen at the top of an annular tank filled with saline water with a stable, linear salinity profile applied a known stress to the fluid and the deepening rate of the mixed layer was measured. The principal problem in this setup is the presence of sidewalls, which take up some of the applied stress. This has to be corrected for. Price (1979) applied these corrections and suggested that the mixed layer depth $D(t) \sim u_* N^{-1/2} t^{1/2}$, where t is time, N is the buoyancy frequency and u_* is the water-side friction velocity, with the proportionality constant being 1.05 (see also Trowbridge 1992). This functional behavior must be reproduced by a turbulence model. We use the KP setup to test the changes brought about by the adoption of new closure constants in the KC model.

Free convection is another limiting case that must be well-simulated by a turbulence model. To do so, we make use of the Mironov et al. (2000) large eddy simulations (LES) of the classic Willis and Deardorff (1974) laboratory experiment on entrainment in a initially linearly stratified fluid heated from below.

Finally, we show the influence oLangmuir cell-driven turbulence and wave breaking on mixing. For this, we choose Station PAPA. Surface wave effects are parameterized as in Kantha and Clayson (2004). The difference between PAPA simulations with and without surface wave effects is small but not insignificant. See Kantha and Clayson (2004) and Carniel et al. (2004) for a more detailed discussion of surface wave effects on the oceanic mixed layer.

5. Concluding Remarks

The Kantha and Clayson (1994) two-equation model has been updated in view of the developments since 1994. A generic two-equation turbulence model is also presented and includes these developments. More importantly, it can simulate traditional two-equation models such as k-ε, k-ω and k-k, as well as non-traditional k-kT, k-T and k-ℓ models.

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